Solutions to the Extra Practice Problems for Chapter 9

1. <u>The velocity is 18.2 m/s south</u>. Equation (9.1) relates momentum, mass, and velocity. Defining south as positive:

$$\mathbf{p} = \mathrm{m} \cdot \mathbf{v}$$

$$39{,}100\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}} = (2{,}145\ \mathrm{kg}) \cdot \mathbf{v}$$

$$\mathbf{v} = \frac{39{,}100\ \frac{\cancel{\mathrm{kg}} \cdot \mathrm{m}}{\mathrm{s}}}{2{,}145\ \cancel{\mathrm{kg}}} = 18.2\ \frac{\mathrm{m}}{\mathrm{s}}$$

The velocity is always in the same direction as the momentum.

2. <u>It takes 0.0012 s to come to rest</u>. Equation (9.4) will allow us to calculate the time, but it needs the change in momentum. The ball starts in one direction and then stops, so I will define the initial direction as positive. Also, we need to use SI units because of the force unit, so I will convert mass to 0.245 kg.

$$\Delta\mathbf{p} = \mathbf{p}_{\mathrm{final}} - \mathbf{p}_{\mathrm{initial}} = 0 - (0.245\ \mathrm{kg}) \cdot \left(12.3\frac{\mathrm{m}}{\mathrm{s}}\right) = -3.01\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

Now we can use Equation (9.4). Remember, however, that the force has a direction. It is obviously opposite the initial direction of the ball, so it is negative.

$$\Delta\mathbf{p} = \mathbf{F} \cdot \Delta\mathrm{t}$$

$$-3.01\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}} = (-2{,}500\ \mathrm{N}) \cdot (\Delta\mathrm{t})$$

$$\Delta\mathrm{t} = \frac{-3.01\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}}{-2{,}500\ \mathrm{N}} = \frac{-3.01\ \frac{\cancel{\mathrm{kg \cdot m}}}{\cancel{\mathrm{s}}}}{-2{,}500\ \frac{\cancel{\mathrm{kg \cdot m}}}{\mathrm{s}^{\cancel{2}}}} = 0.0012\ \mathrm{s}$$

3. <u>Its velocity is 30 m/s in the direction in which the bat hit it</u>. Equation (9.4) will allow us to calculate the change in momentum, which will give us the new velocity. Defining the direction of the force as positive:

$$\Delta\mathbf{p} = \mathbf{F} \cdot \Delta\mathrm{t}$$

$$\Delta\mathbf{p} = (12{,}300\ \mathrm{N}) \cdot (0.00091\ \mathrm{s}) = \left(12{,}300\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}^{\cancel{2}}}\right) \cdot (0.00091\ \cancel{\mathrm{s}}) = 11\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

Now we can use that to get the new velocity. Remember, the force is positive, and it is opposite the initial velocity of the ball, so the initial velocity is negative. Also, we need to convert the mass to 0.143 kg:

$$11\frac{\mathrm{kg \cdot m}}{\mathrm{s}} = \mathbf{p}_{\mathrm{final}} - \mathbf{p}_{\mathrm{initial}}$$

$$11\frac{\mathrm{kg \cdot m}}{\mathrm{s}} = \mathbf{p}_{\mathrm{final}} - (0.143\ \mathrm{kg}) \cdot \left(-49.1\frac{\mathrm{m}}{\mathrm{s}}\right)$$

$$\mathbf{p}_{\mathrm{final}} = 11\frac{\mathrm{kg \cdot m}}{\mathrm{s}} - (0.143\ \mathrm{kg}) \cdot \left(49.1\frac{\mathrm{m}}{\mathrm{s}}\right) = 11\frac{\mathrm{kg \cdot m}}{\mathrm{s}} - 7.02\frac{\mathrm{kg \cdot m}}{\mathrm{s}} = 4\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

Since subtraction goes by decimal place instead of counting significant figures, we needed to round after the multiplication. Then, when we subtracted, the least precise number is 11, so the answer's last significant figure must be in the ones place.

$$\mathbf{p} = \mathrm{m} \cdot \mathbf{v}$$

$$4\,\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}} = (0.143\ \mathrm{kg}) \cdot \mathbf{v}$$

$$\mathbf{v} = \frac{4\,\frac{\cancel{\mathrm{kg}} \cdot \mathrm{m}}{\mathrm{s}}}{0.143\ \cancel{\mathrm{kg}}} = 30\ \frac{\mathrm{m}}{\mathrm{s}}$$

4. <u>The recoil velocity is 0.886 m/s opposite the bullet</u>. It is best to stick with SI units, so let's convert the masses to 0.676 kg and 0.00205 kg. The momentum before shooting is zero, and that must equal the momentum after:

$$\mathbf{p}_{\mathrm{before}} = \mathbf{p}_{\mathrm{after}}$$

$$0 = \mathbf{p}_{\mathrm{bullet}} + \mathbf{p}_{\mathrm{gun}}$$

$$0 = \mathrm{m}_{\mathrm{bullet}} \cdot \mathbf{v}_{\mathrm{bullet}} + \mathrm{m}_{\mathrm{gun}} \cdot \mathbf{v}_{\mathrm{gun}}$$

$$0 = (0.00205\ \mathrm{kg}) \cdot \left(292\,\frac{\mathrm{m}}{\mathrm{s}}\right) + (0.676\ \mathrm{kg}) \cdot \mathbf{v}_{\mathrm{gun}}$$

$$\mathbf{v}_{\mathrm{gun}} = \frac{-(0.00205\ \cancel{\mathrm{kg}}) \cdot \left(292\,\frac{\mathrm{m}}{\mathrm{s}}\right)}{0.676\ \cancel{\mathrm{kg}}} = -0.886\,\frac{\mathrm{m}}{\mathrm{s}}$$

The negative tells us that the gun recoils in the direction opposite the bullet.

5. <u>The bullets have a mass of 0.0246 kg</u>. The momentum before shooting is zero, and that must equal the momentum after:

$$\mathbf{p}_{\mathrm{before}} = \mathbf{p}_{\mathrm{after}}$$

$$0 = \mathbf{p}_{\mathrm{bullet}} + \mathbf{p}_{\mathrm{gun}}$$

$$0 = \mathrm{m}_{\mathrm{bullet}} \cdot \mathbf{v}_{\mathrm{bullet}} + \mathrm{m}_{\mathrm{gun}} \cdot \mathbf{v}_{\mathrm{gun}}$$

Remember, however, that the recoil velocity is opposite the velocity of the bullet, so if we say the bullet's velocity is positive, then the gun's is negative.

$$0 = (\mathrm{m}_{\mathrm{bullet}}) \cdot \left(1{,}050\,\frac{\mathrm{meters}}{\mathrm{s}}\right) + (14.5\ \mathrm{kg}) \cdot \left(-1.78\,\frac{\mathrm{meters}}{\mathrm{s}}\right)$$

$$\mathrm{m}_{\mathrm{bullet}} = \frac{(14.5\ \mathrm{kg}) \cdot 1.78\,\cancel{\frac{\mathrm{meters}}{\mathrm{s}}}}{1{,}050\,\cancel{\frac{\mathrm{meters}}{\mathrm{s}}}} = 0.0246\ \mathrm{kg}$$

6. <u>The velocity is 2.23 m/s in the same direction as the original object's velocity, and it is an inelastic collision</u>. The momentum before catch must equal the momentum after.

$$\mathbf{p}_{\mathrm{before}} = \mathbf{p}_{\mathrm{after}}$$

Defining the direction of the object as positive:

$$m_{astro} \cdot \mathbf{v}_{astro\_before} + m_{object} \cdot \mathbf{v}_{object\_before} = (m_{astro} + m_{object}) \cdot \mathbf{v}_{both}$$

$$(175\text{ kg}) \cdot (0) + (19.7\text{ kg}) \cdot (22.1\frac{\text{m}}{\text{s}}) = (175\text{ kg} + 19.7\text{ kg}) \cdot \mathbf{v}_{both}$$

$$\mathbf{v}_{both} = \frac{(19.7\cancel{\text{kg}}) \cdot (22.1\frac{\text{m}}{\text{s}})}{195\cancel{\text{kg}}} = 2.23\frac{\text{m}}{\text{s}}$$

To determine whether it's elastic or inelastic, we must look at the kinetic energy before and after.

$$KE_{before} = \frac{1}{2}m_{astro} \cdot (v_{astrol})^2 + \frac{1}{2}m_{object} \cdot (\mathbf{v}_{object\_before})^2 = 0 + \frac{1}{2}(19.7\text{ kg}) \cdot \left(22.1\frac{\text{m}}{\text{s}}\right)^2 = 4{,}810\text{ J}$$

$$KE_{after} = \frac{1}{2}(m_{astro} + m_{object}) \cdot (\mathbf{v}_{both})^2 = \frac{1}{2}(195\text{ kg}) \cdot \left(2.23\frac{\text{m}}{\text{s}}\right)^2 = 485\text{ J}$$

The energy after is much lower, so this is an inelastic collision.

7. <u>The second ball's velocity is 0.5 m/s opposite of its original direction, and this is inelastic</u>. The momentum before the collision must equal the momentum after.

$$\mathbf{p}_{before} = \mathbf{p}_{after}$$

Defining the initial direction of the first object as positive makes the initial velocity of the second object negative, and the velocity of the first object afterwards also negative:

$$m_1 \cdot \mathbf{v}_{1\_before} + m_2 \cdot \mathbf{v}_{2\_before} = m_1 \cdot \mathbf{v}_{1\_after} + m_2 \cdot \mathbf{v}_{2\_after}$$

$$(1.7\text{ kg}) \cdot (3.2\frac{\text{m}}{\text{s}}) + (2.2\text{ kg}) \cdot (-4.8\frac{\text{m}}{\text{s}}) = (1.7\text{ kg}) \cdot (-3.8\frac{\text{m}}{\text{s}}) + (2.2\text{ kg}) \cdot \mathbf{v}_{2\_after}$$

$$5.4\frac{\text{kg} \cdot \text{m}}{\text{s}} - 11\frac{\text{kg} \cdot \text{m}}{\text{s}} = -6.5\frac{\text{kg} \cdot \text{m}}{\text{s}} + (2.2\text{ kg}) \cdot \mathbf{v}_{2\_after}$$

$$5.4\frac{\text{kg} \cdot \text{m}}{\text{s}} - 11\frac{\text{kg} \cdot \text{m}}{\text{s}} + 6.5\frac{\text{kg} \cdot \text{m}}{\text{s}} = (2.2\text{ kg}) \cdot \mathbf{v}_{2\_after}$$

$$\mathbf{v}_{2\_after} = \frac{1\frac{\cancel{\text{kg}} \cdot \text{m}}{\text{s}}}{2.2\cancel{\text{kg}}} = 0.5\frac{\text{m}}{\text{s}}$$

Since addition and subtraction are done by decimal place, value on the left side of the equation must have its last significant figure in the ones place, which gives you only one significant figure for the velocity. To determine elastic or inelastic, you must look at the kinetic energy before and after:

$$KE_{before} = \frac{1}{2}m_1 \cdot (v_{1\_before})^2 + \frac{1}{2}m_2 \cdot (v_{2\_before})^2 = \frac{1}{2}(1.7\text{ kg}) \cdot \left(3.2\frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(2.2\text{ kg}) \cdot \left(4.8\frac{\text{m}}{\text{s}}\right)^2 = 34.0\text{ J}$$

$$KE_{after} = \frac{1}{2}m_1 \cdot (v_{1\_after})^2 + \frac{1}{2}m_2 \cdot (v_{2\_after})^2 = \frac{1}{2}(1.7\text{ kg}) \cdot \left(3.8\frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(2.2\text{ kg}) \cdot \left(0.5\frac{\text{m}}{\text{s}}\right)^2 = 12\text{ J}$$

The energy after is much lower, so this is an inelastic collision.

8. <u>Bullets leave the gun at 190 m/s</u>. The height tells you the maximum potential energy. At that point, the bullet and pendulum are together, so the mass is 0.915 kg + 0.00723 kg = 0.922 kg.

$$PE = m \cdot g \cdot h = (0.922\ kg) \cdot \left(9.81\ \frac{m}{s^2}\right) \cdot (0.12\ m) = 1.1\ J$$

That is also the initial kinetic energy of the bullet/pendulum:

$$KE = \frac{1}{2} m \cdot v^2$$

$$1.1\ J = \frac{1}{2}(0.922\ kg) \cdot (v)^2$$

$$v = \sqrt{\frac{2 \cdot (1.1\ J)}{0.922\ kg}} == \sqrt{\frac{2 \cdot (1.1\ \frac{\cancel{kg} \cdot m^2}{s^2})}{0.922\ \cancel{kg}}} = 1.5\ \frac{m}{s}$$

That velocity can be used in the momentum conservation equation to get the v of the bullet

$$\mathbf{p}_{before} = \mathbf{p}_{after}$$

$$m_{bullet} \cdot \mathbf{v}_{bullet} = m_{block/bullet\ system} \cdot \mathbf{v}_{block/bullet\ system}$$

$$(0.00723\ kg) \cdot (\mathbf{v}_{bullet}) = (0.922\ kg) \cdot (1.5\ \frac{m}{s})$$

$$\mathbf{v}_{bullet} = \frac{(0.922\ \cancel{kg}) \cdot (1.5\ \frac{m}{s})}{0.00723\ \cancel{kg}} = 190\ \frac{m}{s}$$

9. <u>It will reach a maximum height of 1.6 m</u>. In this case, we know the velocity of the bullet, so we can use that to get the velocity of the bullet+pendulum, remembering to add the masses to get the total mass of the block/bullet system:

$$\mathbf{p}_{before} = \mathbf{p}_{after}$$

$$m_{bullet} \cdot \mathbf{v}_{bullet} = m_{block/bullet\ system} \cdot \mathbf{v}_{block/bullet\ system}$$

$$(0.015\ kg) \cdot (292\ \frac{m}{s}) = (0.787\ kg) \cdot (\mathbf{v}_{block/bullet\ system})$$

$$\mathbf{v}_{block/bullet\ system} = \frac{(0.015\ \cancel{kg}) \cdot (292\ \frac{m}{s})}{0.787\ \cancel{kg}} = 5.6\ \frac{m}{s}$$

That allows us to get the kinetic energy of the bullet+pendulum:

$$KE = \frac{1}{2} m \cdot v^2 = \frac{1}{2}(0.787\ kg) \cdot \left(5.6 \frac{m}{s}\right)^2 = 12 \frac{kg \cdot m^2}{s^2}$$

That all gets converted to PE at the maximum height:

$$PE = m \cdot g \cdot h$$

$$12\ \frac{kg \cdot m^2}{s^2} = (0.787\ kg) \cdot \left(9.81\ \frac{m}{s^2}\right) \cdot h$$

$$h = \frac{12\ \frac{\cancel{kg} \cdot m^{\cancel{2}}}{\cancel{s^2}}}{(0.787\ \cancel{kg}) \cdot \left(9.81\ \frac{\cancel{m}}{\cancel{s^2}}\right)} = 1.6\ m$$

10. The new speed is 9.0×10^2 m/s. Equation (9.7) gives us the angular momentum in this situation.

$$\cancel{\mathrm{m}} \cdot \mathrm{v_{before}} \cdot \mathrm{r_{before}} = \cancel{\mathrm{m}} \cdot \mathrm{v_{after}} \cdot \mathrm{r_{after}}$$

$$\left(1{,}456\ \frac{\text{meters}}{\text{s}}\right) \cdot (1.3\ \text{meters}) = (\mathrm{v_{after}}) \cdot (2.1\ \text{meters})$$

$$\mathrm{v_{after}} = \frac{\left(1{,}456\ \frac{\text{meters}}{\text{s}}\right) \cdot (1.3\ \cancel{\text{meters}})}{2.1\ \cancel{\text{meters}}} = 9.0 \times 10^2\ \frac{\text{meters}}{\text{s}}$$