Solutions to the Extra Practice Problems for Chapter 10

1. The mass is 0.645 kg. We can figure out the force the spring is using with Hooke's Law, once we convert the distance to 0.117 meters. Also, since the displacement is down, it's negative:

$$\mathbf{F} = -\mathrm{k} \cdot \Delta \mathbf{x}$$

$$\mathbf{F} = -\left(54.1\ \frac{\mathrm{N}}{\cancel{\mathrm{m}}}\right) \cdot (-0.117\ \cancel{\mathrm{m}}) = 6.33\ \mathrm{N}$$

But that's the weight, not the mass. To get the mass:

$$\text{weight} = \mathrm{m} \cdot \mathrm{g}$$

$$6.33\ \mathrm{N} = \mathrm{m} \cdot \left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)$$

$$\mathrm{m} = \frac{6.33\ \mathrm{N}}{9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}} = \frac{6.33\ \frac{\mathrm{kg} \cdot \cancel{\mathrm{m}}}{\cancel{\mathrm{s}^2}}}{9.81\ \frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}^2}}} = 0.645\ \mathrm{kg}$$

2. The spring constant is 3.9 N/m. We need to know the weight to determine the force that the spring must exert. To do that, though, mass needs to be converted to 0.0150 kg.

$$\text{weight} = \mathrm{m} \cdot \mathrm{g} = (0.0150\ \mathrm{kg}) \cdot \left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) = 0.147\ \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} = 0.147\ \mathrm{N}$$

That means the spring pulls up with a force of 0.147 N. To use Hooke's Law, however, the distance must be converted to 0.038 m, and it is negative, because the spring stretches down.

$$\mathbf{F} = -\mathrm{k} \cdot \Delta \mathbf{x}$$

$$0.147\ \mathrm{N} = -\mathrm{k} \cdot (-0.038\ \mathrm{m})$$

$$\mathrm{k} = \frac{0.147\ \mathrm{N}}{0.038\ \mathrm{m}} = 3.9\ \frac{\mathrm{N}}{\mathrm{m}}$$

3. The period is 0.277 s. We will need the force constant to get the period. That comes from Hooke's Law, which requires the force the spring uses, which is based on the weight, not the mass. Converting mass to 0.199 kg:

$$\text{weight} = \mathrm{m} \cdot \mathrm{g} = (0.199\ \mathrm{kg}) \cdot \left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) = 1.95\ \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} = 1.95\ \mathrm{N}$$

That weight produces a downward stretch of 0.0192 m, which will now allow us to get the force constant:

$$\mathbf{F} = -\mathrm{k} \cdot \Delta \mathbf{x}$$

$$1.95\ \mathrm{N} = -\mathrm{k} \cdot (-0.0192\ \mathrm{m})$$

$$\mathrm{k} = \frac{1.95\ \mathrm{N}}{0.0192\ \mathrm{m}} = 102\ \frac{\mathrm{N}}{\mathrm{m}}$$

That force constant and the mass will allow us to determine the period:

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}} = 2\pi\sqrt{\frac{0.199\ \mathrm{kg}}{102\ \frac{\mathrm{N}}{\mathrm{m}}}} = 2\pi\sqrt{\frac{0.199\ \cancel{\mathrm{kg}}}{102\ \frac{\frac{\cancel{\mathrm{kg}} \cdot \cancel{\mathrm{m}}}{\mathrm{s}^2}}{\cancel{\mathrm{m}}}}} = 0.277\ \mathrm{s}$$

4. The force constant is 60.8 N/m. The period can give us the force constant:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$3.12\ \mathrm{s} = 2\pi\sqrt{\frac{15.0\ \mathrm{kg}}{k}}$$

$$(3.12\ \mathrm{s})^2 = (2\pi)^2 \cdot \left(\frac{15.0\ \mathrm{kg}}{k}\right)$$

$$k = (2\pi)^2 \cdot \left(\frac{15.0\ \mathrm{kg}}{(3.12\ \mathrm{s})^2}\right) = 60.8\ \frac{\mathrm{kg}}{\mathrm{s}^2} = 60.8\ \frac{\mathrm{N}}{\mathrm{m}}$$

5. The mass is 56 kg. The period can give us the mass:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$5.0\ \mathrm{s} = 2\pi\sqrt{\frac{m}{89\ \frac{\mathrm{N}}{\mathrm{m}}}}$$

$$(5.0\ \mathrm{s})^2 = (2\pi)^2 \cdot \left(\frac{m}{89\ \frac{\mathrm{N}}{\mathrm{m}}}\right)$$

$$m = \frac{(5.0\ \mathrm{s})^2}{(2\pi)^2} \cdot \left(89\ \frac{\mathrm{N}}{\mathrm{m}}\right) = \frac{25\ \cancel{\mathrm{s}^2}}{(2\pi)^2} \cdot \left(89\ \frac{\frac{\mathrm{kg}\cdot\cancel{\mathrm{m}}}{\cancel{\mathrm{s}^2}}}{\cancel{\mathrm{m}}}\right) = 56\ \mathrm{kg}$$

6. The spring will compress 6.09 m. The mass has kinetic energy before it hits the spring.

$$KE = \frac{1}{2}m \cdot v^2 = \frac{1}{2}(2.84\ \mathrm{kg}) \cdot \left(45.1\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = 2{,}890\ \mathrm{J}$$

The mass will stop when all of that kinetic energy is converted into potential energy of the spring:

$$PE = \frac{1}{2}k \cdot \Delta x^2$$

$$2{,}890\ \mathrm{J} = \frac{1}{2}\left(156\ \frac{\mathrm{N}}{\mathrm{m}}\right) \cdot \Delta x^2$$

$$\Delta x = \sqrt{\frac{2 \cdot 2{,}890\ \mathrm{J}}{156\ \frac{\mathrm{N}}{\mathrm{m}}}} = \sqrt{\frac{2 \cdot 2{,}890\ \cancel{\mathrm{N}} \cdot \mathrm{m}}{156\ \frac{\cancel{\mathrm{N}}}{\mathrm{m}}}} = 6.09\ \mathrm{m}$$

7. The maximum speed is 0.467 m/s. The spring's total energy can be determined, but the amplitude must be converted to 0.143 m:

$$TE = \frac{1}{2}k \cdot A^2 = \frac{1}{2}\left(65.2\ \frac{\mathrm{N}}{\mathrm{m}}\right) \cdot (0.143\ \mathrm{m})^2 = 0.667\ \mathrm{N}\cdot\mathrm{m} = 0.667\ \mathrm{J}$$

At any given point, that is the sum of the kinetic and potential energy. The maximum speed will be when all of that energy is kinetic:

$$\mathrm{KE} = \frac{1}{2}\mathrm{m} \cdot \mathrm{v}^2$$

$$0.667\ \mathrm{J} = \frac{1}{2}(6.12\ \mathrm{kg}) \cdot \mathrm{v}^2$$

$$\mathrm{v} = \sqrt{\frac{2 \cdot 0.667\ \mathrm{J}}{6.12\ \mathrm{kg}}} = \sqrt{\frac{2 \cdot 0.667\ \frac{\cancel{\mathrm{kg}} \cdot \mathrm{m}^2}{\mathrm{s}^2}}{6.12\ \cancel{\mathrm{kg}}}} = 0.467\ \frac{\mathrm{m}}{\mathrm{s}}$$

8. <u>The speed will be 0.424 m/s</u>. We already determined that the total energy is 0.667 J, and that will be the sum of the potential and kinetic energies. Converting the displacement to 0.0600 m:

$$\frac{1}{2}\mathrm{k} \cdot \Delta \mathrm{x}^2 + \frac{1}{2}\mathrm{m} \cdot \mathrm{v}^2 = 0.667\ \mathrm{N} \cdot \mathrm{m}$$

$$\frac{1}{2}\left(65.2\ \frac{\mathrm{N}}{\mathrm{m}}\right) \cdot (0.0600\ \mathrm{m})^2 + \frac{1}{2}(6.12\ \mathrm{kg}) \cdot \mathrm{v}^2 = 0.667\ \mathrm{N} \cdot \mathrm{m}$$

$$0.117\ \mathrm{N} \cdot \mathrm{m} + \frac{1}{2}(6.12\ \mathrm{kg}) \cdot \mathrm{v}^2 = 0.667\ \mathrm{N} \cdot \mathrm{m}$$

$$\frac{1}{2}(6.12\ \mathrm{kg}) \cdot \mathrm{v}^2 = 0.550\ \mathrm{N} \cdot \mathrm{m}$$

$$\mathrm{v} = \sqrt{\frac{2 \cdot 0.550\ \mathrm{J}}{6.12\ \mathrm{kg}}} = \sqrt{\frac{2 \cdot 0.550\ \frac{\cancel{\mathrm{kg}} \cdot \mathrm{m}^2}{\mathrm{s}^2}}{6.12\ \cancel{\mathrm{kg}}}} = 0.424\ \frac{\mathrm{m}}{\mathrm{s}}$$

9. <u>The period is 1.51 s</u>. The period is given by Equation (10.11), but the length must be converted to 0.564 m.

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{L}}{\mathrm{g}}} = 2\pi\sqrt{\frac{0.564\ \mathrm{m}}{9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}}} = 1.51\ \mathrm{s}$$

10. <u>The acceleration due to the artificial gravity is a whopping 110 m/s^2</u>. We can use the same equation to get the acceleration due to the artificial gravity:

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{L}}{\mathrm{g}}}$$

$$0.45 = 2\pi\sqrt{\frac{(0.564\ \mathrm{m})}{\mathrm{g}}}$$

$$(0.45\ \mathrm{s})^2 = (2\pi)^2\left(\frac{(0.564\ \mathrm{m})}{\mathrm{g}}\right)$$

$$\mathrm{g} = (2\pi)^2\left(\frac{(0.564\ \mathrm{m})}{(0.45\ \mathrm{s})^2}\right) = 110\ \frac{\mathrm{m}}{\mathrm{s}^2}$$