Solutions to the Extra Practice Problems for Chapter 7

1. The radius is 0.658 m. We can figure out the radius using Equation (7.2), but for that we need the period:

$$f = \frac{1}{T}$$

$$1.09\ \text{Hz} = \frac{1}{T}$$

$$T = \frac{1}{1.09\ \frac{1}{s}} = 0.917\ s$$

We can now determine the radius:

$$v = \frac{2 \cdot \pi \cdot r}{T}$$

$$4.51\ \frac{m}{s} = \frac{2 \cdot (3.142) \cdot r}{0.917\ s}$$

$$r = \frac{(4.51\ \frac{m}{\cancel{s}}) \cdot (0.917\ \cancel{s})}{2 \cdot (3.142)} = 0.658\ m$$

I used four significant figures for pi, but you could have used three and gotten 0.659 m, which is equivalent to my answer, since there is always error in the last significant figure. You should not use two (3.1), since that reduces the significant figures in the final answer.

2. The speed is 3.5 m/s. The centripetal force is given by Equation (7.3), but the mass needs to be converted to 0.734 kg, and the radius must be converted to 0.62 m, since Newtons uses kg and m:

$$F_c = \frac{m \cdot v^2}{r}$$

$$14.2\ N = \frac{0.734\ kg \cdot v^2}{0.62\ m}$$

$$v = \sqrt{\frac{(14.2\ N) \cdot 0.62\ m}{0.734\ kg}} = \sqrt{\frac{(14.2\ \frac{\cancel{kg} \cdot m}{s^2}) \cdot 0.62\ m}{0.734\ \cancel{kg}}} = 3.5\ \frac{m}{s}$$

3. The centripetal force is divided by 4 when the speed is cut in half, and it is multiplied by 2 when the radius is cut in half. Equation (7.4) says that acceleration depends on v^2. Thus, whatever you do to v, you must do the same thing squared to F. It also says that acceleration depends on the inverse of r. Thus, whatever you do to r, you invert and do it to F.

4. The maximum mass is 0.29 kg. Since centripetal force increases with increasing mass, we can set the maximum tension equal to the centripetal force in Equation (7.3) and solve for m. Anything larger will require a larger force and thus break the thread. To use the equation, though, the radius must be converted to 0.72 m.

$$F_c = \frac{m \cdot v^2}{r}$$

$$59.5 \text{ N} = \frac{m \cdot (12.1 \frac{\text{meters}}{\text{s}})^2}{0.72 \text{ meters}}$$

$$m = \frac{(59.5 \text{ N}) \cdot (0.72 \text{ meters})}{(12.1 \frac{\text{meters}}{\text{s}})^2} = \frac{(59.5 \frac{\text{kg} \cdot \cancel{\text{meters}}}{\cancel{\text{s}^2}}) \cdot (0.72 \cancel{\text{meters}})}{146.41 \frac{\cancel{\text{meters}^2}}{\cancel{\text{s}^2}}} = 0.29 \text{ kg}$$

I didn't round when I squared 12.1, since everything is multiplication and division.

5. The minimum radius of curvature is 670 m. Static friction supplies the centripetal force for this kind of circular motion, so we need to set the equation for static friction equal to Equation (7.3):

$$\mu \cdot \cancel{m} \cdot g = \frac{\cancel{m} \cdot v^2}{r}$$

$$\mu \cdot g = \frac{v^2}{r}$$

Now we can put in the numbers, remembering that the static friction coefficient is the larger one:

$$(0.41) \cdot (9.81 \frac{\text{m}}{\text{s}^2}) = \frac{(52 \frac{\text{m}}{\text{s}})^2}{r}$$

$$r = \frac{(52 \frac{\text{m}}{\text{s}})^2}{(0.41) \cdot (9.81 \frac{\text{m}}{\text{s}^2})} = \frac{2704 \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{(0.41) \cdot (9.81 \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}})} = 670 \text{ m}$$

Since the centripetal force increases for decreasing radius, anything smaller than that will require more centripetal force than static friction will allow.

6. The distance between the centers is 0.885 m. The gravitational force is given by Equation (7.5):

$$F_g = \frac{G \cdot m_1 \cdot m_2}{r^2}$$

$$6.92 \times 10^{-10} \text{ N} = \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \cdot (2.85 \text{ kg}) \cdot (2.85 \text{ kg})}{r^2}$$

$$r = \sqrt{\frac{(6.67 \times 10^{-11} \frac{\cancel{\text{N}} \cdot \text{m}^2}{\cancel{\text{kg}^2}}) \cdot (2.85 \cancel{\text{kg}}) \cdot (2.85 \cancel{\text{kg}})}{6.92 \times 10^{-10} \cancel{\text{N}}}} = 0.885 \text{ m}$$

7. The force is divided by 4 when one mass is divided by 4, and it is multiplied by 16 when the distance is divided by 4. Equation (7.5) says that the force depends on mass, so whatever you do to the mass, you must do to the force. However, the force depends on $1/r^2$, so whatever you do to the distance, you invert, square, and then do to the force.

8. The gravitational force is 150 N. Near the surface of the earth (at heights under 500 m), the height is insignificant compared to the radius of the earth. Thus, the distance between the centers of the object and the earth stay constant to within the kinds of precisions we can measure, so the force is constant.

9. The speed is 4,300 m/s. The centripetal force is given by Equation (7.3), and the gravitational force by Equation (7.5). Since gravity is supplying the centripetal force, we set them equal to one another. Remember that in Equation (7.3), the m is of the thing that is moving. In this case, that's the satellite. Also, remember that radius must be in meters, so it's 2.2×10^7 m.

$$\frac{\cancel{m_{satellite}} \cdot v^2}{\cancel{r}} = \frac{G \cdot \cancel{m_{satellite}} \cdot m_{earth}}{r^{\cancel{2}}}$$

$$v^2 = \frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}) \cdot (5.97 \times 10^{24}\ kg)}{2.2 \times 10^7\ m}$$

$$v = \sqrt{\frac{(6.67 \times 10^{-11} \frac{\frac{\cancel{kg} \cdot \cancel{m}}{s^2} \cdot m^2}{\cancel{kg^2}}) \cdot (5.97 \times 10^{24}\ \cancel{kg})}{2.2 \times 10^7\ \cancel{m}}} = 4{,}300\ \frac{m}{s}$$

10. The mass is 1.9×10^{32} kg. Setting the equations for gravitational and centripetal force equal once again, we get the same basic equation we got in 9:

$$v^2 = \frac{G \cdot m_{star}}{r}$$

To get the mass of the star, then, we need its speed, which we can get from Equation (7.2), but the period needs to be converted to 5,800,000 s, and r needs to be converted to 2.2×10^{11} m.

$$v = \frac{2 \cdot \pi \cdot r}{T} = \frac{2 \cdot (3.14) \cdot (2.2 \times 10^{11} m)}{5{,}800{,}000\ s} = 2.4 \times 10^5\ \frac{m}{s}$$

Now we can use the equation:

$$(2.4 \times 10^5\ \frac{m}{s})^2 = \frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}) \cdot m_{star}}{2.2 \times 10^{11}\ m}$$

$$m_{star} = \frac{(2.4 \times 10^5\ \frac{m}{s})^2 \cdot (2.2 \times 10^{11}\ m)}{6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}} = \frac{(5.76 \times 10^{10}\ \frac{\cancel{m^2}}{\cancel{s^2}}) \cdot (2.2 \times 10^{11} \cancel{m})}{6.67 \times 10^{-11} \frac{\frac{\cancel{kg} \cdot \cancel{m}}{\cancel{s^2}} \cdot \cancel{m^2}}{kg^{\cancel{2}}}} = 1.9 \times 10^{32}\ kg$$