Solutions to the Extra Practice Problems for Chapter 6

1. The angle is 51° and the mass is 0.92 kg. For T_1, we use the fact that when two parallel lines are cut by a transversal, alternating interior angles are congruent. However, that still doesn't give us the physics angle for T_1. The physics angle adds to that angle to make exactly 180°, so it is 139°. For T_2, the alternating angle is also the physics angle, so its angle is θ.

$$T_1 \cdot \cos(139^o) + T_2 \cdot \cos(\theta) + w \cdot \cos(270^o) = 0$$

$$(5.7\text{ N}) \cdot \cos(139^o) + (6.8\text{ N}) \cdot \cos(\theta) = 0$$

$$\cos(\theta) = \frac{-(5.7\text{ N}) \cdot \cos(139^o)}{6.8\text{ N}} = 0.63$$

$$\theta = \cos^{-1}(0.63) = 51^o$$

In the y-dimension:

$$T_1 \cdot \sin(139^o) + T_2 \cdot \sin(\theta) + w \cdot \sin(270^o) = 0$$

$$(5.7\text{ N}) \cdot \sin(139^o) + (6.8\text{ N}) \cdot \sin(51^o) - w = 0$$

$$w = (5.7\text{ N}) \cdot \sin(139^o) + (6.8\text{ N}) \cdot \sin(51^o) = 3.7\text{ N} + 5.3\text{ N} = 9.0\text{ N}$$

But that's the weight, which is m·g. Thus, we divide by g to get the mass, which is 0.92 kg.

2. The coefficient of kinetic friction is 0.253. When it is given a shove, kinetic friction takes over, and if the velocity is constant, not matter what its value, Equation (6.6) applies and gives us the coefficient of kinetic friction.

$$\mu = \tan(\theta) = \tan(14.2^o) = 0.253$$

3. The coefficient of static friction is 0.475. This is just like the experiment. Equation (6.6) will give us the coefficient of kinetic friction.

$$\mu = \tan(\theta) = \tan(25.4^o) = 0.475$$

4. The coefficient of kinetic friction is 0.413. Since the box is accelerating, Equation (6.6) doesn't apply. Thus, we must sum up the forces, defining down the ramp as negative:

$$-\cancel{m} \cdot g \cdot \sin(\theta) + \mu_k \cdot \cancel{m} \cdot g \cdot \cos(\theta) = \cancel{m} \cdot \mathbf{a}$$

$$-g \cdot \sin(\theta) + \mu_k \cdot g \cdot \cos(\theta) = \mathbf{a}$$

We can now plug in the values we know, but remember that down the incline is defined as negative. Thus, the acceleration is negative, not positive.

$$-\left(9.81\frac{\text{m}}{\text{s}^2}\right) \cdot \sin(25.4^o) + \mu_k \cdot \left(9.81\frac{\text{m}}{\text{s}^2}\right) \cdot \cos(25.4^o) = -0.55\frac{\text{m}}{\text{s}^2}$$

$$\mu_k \cdot \left(9.81\frac{\text{m}}{\text{s}^2}\right) \cdot \cos(25.4^o) = -0.55\frac{\text{m}}{\text{s}^2} + 4.21\frac{\text{m}}{\text{s}^2}$$

$$\mu_k = \frac{3.66\cancel{\frac{\text{m}}{\text{s}^2}}}{\left(9.81\cancel{\frac{\text{m}}{\text{s}^2}}\right) \cdot \cos(25.4^o)} = 0.413$$

5. The acceleration is 3.2 m/s² down the ramp. Whether it is moving up or down the box, it is moving, so only kinetic friction plays a role. Thus, we use the smaller coefficient of friction. Defining down the ramp as negative:

$$-\cancel{m}\cdot g\cdot \sin(\theta)+\mu_k\cdot \cancel{m}\cdot g\cdot \cos(\theta)=\cancel{m}\cdot \mathbf{a}$$

$$-g\cdot \sin(\theta)+\mu_k\cdot g\cdot \cos(\theta)=\mathbf{a}$$

We can now plug in the values we know:

$$-\left(9.81\frac{\text{m}}{\text{s}^2}\right)\cdot \sin(33.0^o)+(0.25)\cdot\left(9.81\frac{\text{m}}{\text{s}^2}\right)\cdot \cos(33.0^o)=\mathbf{a}$$

$$\mathbf{a}=-5.34\ \frac{\text{m}}{\text{s}^2}+2.1\ \frac{\text{m}}{\text{s}^2}=-3.2\ \frac{\text{m}}{\text{s}^2}$$

6. The acceleration is 3.3 m/s², and the tension in the string is 0.85 N. We can look at each box individually. For the first box, the force is to the right, which I will call positive. That means T is negative. Also, note that since force is in N, mass must be in kg:

$$1.2\text{ N}+-\text{T}=(0.112\text{ kg})\cdot(\mathbf{a})$$

We have two unknowns, so looking at the second box, the only force acting on it is the tension in the string, which pulls to the right. That makes it positive.

$$\text{T}=(0.257\text{ kg})\cdot(\mathbf{a})$$

We can plug that in for T in the first equation:

$$1.2\text{ N}-(0.257\text{ kg})\cdot(\mathbf{a})=(0.112\text{ kg})\cdot(\mathbf{a})$$

$$(0.369\text{ kg})\cdot\mathbf{a}=1.2\text{ N}$$

$$\mathbf{a}=\frac{1.2\text{ N}}{0.369\text{ kg}}=\frac{1.2\ \frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}}{0.369\ \cancel{\text{kg}}}=3.3\frac{\text{m}}{\text{s}^2}$$

Now we can get tension:

$$\text{T}=(0.257\text{ kg})\cdot(\mathbf{a})=(0.257\text{ kg})\cdot\left(3.3\frac{\text{m}}{\text{s}^2}\right)=0.85\text{ N}$$

7. The maximum force is 2.9 N. Since T = 2.0 N is the maximum value the string can hold, the first box's equation becomes:

$$\text{F}+-2.0\text{ N}=(0.112\text{ kg})\cdot(\mathbf{a})$$

We have two unknowns, so looking at the second box, the only force acting on it is the tension in the chain, which pulls to the right. That makes it positive.

$$2.0\text{ N}=(0.257\text{ kg})\cdot(\mathbf{a})$$

$$\mathbf{a}=\frac{2.0\text{ N}}{0.257\text{ kg}}=\frac{2.0\ \frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}}{0.257\ \cancel{\text{kg}}}=7.8\frac{\text{m}}{\text{s}^2}$$

Putting that into the first equation:

$$F + -2.0\text{ N} = (0.112\text{ kg}) \cdot (7.8\frac{\text{m}}{\text{s}^2})$$

$$F = 0.87\text{ N} + 2.0\text{ N} = 2.9\text{ N}$$

8. No, it is not in rotational equilibrium. For that to happen, the sum of the torques must be zero. That's not possible here, since the second force is *parallel* to the lever arm. Thus, there is no perpendicular component to that force, so the torque it exerts is zero. Thus, there is only one force making a torque, which means the sum cannot be zero.

9. The mass is 1.4 kg. The 1.9-kg plane produces clockwise motion, so its torque is negative.

$$\tau_{\text{unknown}} + -\tau_{1.9\text{kg}} = 0$$

$$(\text{m}) \cdot \cancel{g} \cdot (9.8\text{ cm}) - (1.9\text{ kg}) \cdot \cancel{g} \cdot (7.2\text{ cm}) = 0$$

$$\text{m} = \frac{(1.9\text{ kg}) \cdot (7.2\ \cancel{\text{cm}})}{9.8\ \cancel{\text{cm}}} = 1.4\text{ kg}$$

10. The plane must be 1.71 cm right of the axis of rotation. The planes on the right have a negative torque, while the one on the left is positive

$$\tau_{556\text{g}} + -\tau_{702\text{g}} + -\tau_{519\text{g}} = 0$$

$$(556\text{ grams}) \cdot \cancel{g} \cdot (11.5\text{ cm}) - (702\text{ grams}) \cdot \cancel{g} \cdot (r_{702\text{g}}) - (519\text{grams}) \cdot \cancel{g} \cdot (10.0\text{ cm}) = 0$$

$$r_{702\text{g}} = \frac{(556\text{ grams}) \cdot (11.5\text{ cm}) - (519\text{ grams}) \cdot (10.0\text{ cm})}{702\text{ grams}}$$

$$r_{7.5\text{g}} = \frac{6{,}390\text{ gram} \cdot \text{cm} - 5{,}190\text{ gram} \cdot \text{cm}}{702\text{ grams}} = \frac{1.20 \times 10^3\ \cancel{\text{gram}} \cdot \text{cm}}{702\ \cancel{\text{grams}}} = 1.71\text{ cm}$$