Solutions to the Extra Practice Problems for Chapter 5

1. The final displacement is $\underline{2.0\text{x}10^3 \text{ m at } 1.50\text{x}10^2\ ^o}$. First, we have to turn the velocity vectors into displacement vectors so they can be added. To do that, we need to make the time units consistent:

$$\frac{12.0\ \cancel{\text{min}}}{1} \times \frac{60\text{ s}}{1\ \cancel{\text{min}}} = 7.20\times10^2\text{ s}$$

$$\frac{10.7\ \cancel{\text{min}}}{1} \times \frac{60\text{ s}}{1\ \cancel{\text{min}}} = 642\text{ s}$$

Now we can use Equation (2.16) to calculate the displacement in each segment of the journey:

$$\text{Segment 1: } \Delta\mathbf{x} = \mathbf{v}_0\text{t} + \frac{1}{2}\mathbf{a}\text{t}^2 = \left(1.9\frac{\text{m}}{\cancel{\text{s}}}\right)\cdot(7.20\times10^2\ \cancel{\text{s}}) = 1{,}400\text{ m}$$

$$\text{Segment 2: } \Delta\mathbf{x} = \mathbf{v}_0\text{t} + \frac{1}{2}\mathbf{a}\text{t}^2 = \left(1.7\frac{\text{m}}{\cancel{\text{s}}}\right)\cdot(642\ \cancel{\text{s}}) = 1{,}100\text{ m}$$

The angle for each displacement will be the same as the angle for the velocity in that segment. Calling the first displacement vector **A**, the second one **B**, and their sum **C**:

$$\text{A}_\text{x} = \text{A}\cdot\cos(\theta) = (1{,}400\text{ m})\cdot\cos(117^\text{o}) = -640\text{ m}$$

$$\text{A}_\text{y} = \text{A}\cdot\sin(\theta) = (1{,}400\text{ m})\cdot\sin(117^\text{o}) = 1{,}200\text{ m}$$

$$\text{B}_\text{x} = \text{B}\cdot\cos(\theta) = (1{,}100\text{ m})\cdot\cos(193^\text{o}) = -1{,}100\text{ m}$$

$$\text{B}_\text{y} = \text{B}\cdot\sin(\theta) = (1{,}100\text{ m})\cdot\sin(193^\text{o}) = -250\text{ m}$$

Now we can add the x- and y-components together:

$$\text{C}_\text{x} = \text{A}_\text{x} + \text{B}_\text{x} = -640\text{ m} + -1{,}100\text{ m} = -1{,}700\text{ m}$$

$$\text{C}_\text{y} = \text{A}_\text{y} + \text{B}_\text{y} = 1{,}200\text{ m} + -250\text{ m} = 1.0\times10^3\text{ m}$$

Now that we have the components of the vector sum, we can construct the final velocity vector:

$$\text{C} = \sqrt{\text{C}_\text{x}^2 + \text{C}_\text{y}^2} = \sqrt{(-1{,}700\text{ m})^2 + (1.0\times10^3\text{ m})^2} = \sqrt{2{,}900{,}000\text{ m}^2 + 1.0\times10^6\text{ m}^2} = \sqrt{3{,}900{,}000\text{ m}^2}$$

$$\text{C} = 2.0\times10^3\text{ m}$$

When we square, both numbers have their last significant figure in the hundred thousands place (the 0 in $1.0\text{x}10^6$ is in the hundred thousands place), so the sum must as well. That gives two significant figures for the square root. Now we can calculate the angle:

$$\tan(\theta) = \frac{\text{C}_\text{y}}{\text{C}_\text{x}}$$

$$\tan(\theta) = \frac{1.0\times10^3\ \cancel{\text{m}}}{-1{,}700\ \cancel{\text{m}}}$$

$$\tan(\theta) = -0.5882\ldots$$

$$\theta = \tan^{-1}(-0.5882\ldots) = -3.0\times10^{1\,\text{o}}$$

Since the x-component is negative and the y-component is positive, it is quadrant II, so we must add exactly 180°.

$$\theta = -3.0 \times 10^{1^{o}} + 180^{o} = 1.50 \times 10^{2^{o}}$$

2. <u>The final displacement is 35 km at 56°</u>. In this problem, we have a ship in a current, so the two velocities add to give the actual velocity. We can then use that velocity and the time to determine the final displacement. Calling the velocity given by the engines vector **A**, the velocity of the current vector **B**, and the sum vector **C**:

$$A_x = A \cdot \cos(\theta) = \left(21 \frac{km}{hr}\right) \cdot \cos(58^{o}) = 11 \frac{km}{hr}$$

$$A_y = A \cdot \sin(\theta) = \left(21 \frac{km}{hr}\right) \cdot \sin(58^{o}) = 18 \frac{km}{hr}$$

$$B_x = B \cdot \cos(\theta) = \left(3.2 \frac{km}{hr}\right) \cdot \cos(251^{o}) = -1.0 \frac{km}{hr}$$

$$B_y = B \cdot \sin(\theta) = \left(3.2 \frac{km}{hr}\right) \cdot \sin(251^{o}) = -3.0 \frac{km}{hr}$$

Now we can add the x-components and y-components. The results will be the x- and y-components of the vector sum:

$$C_x = A_x + B_x = 11 \frac{km}{hr} - 1.0 \frac{km}{hr} = 1.0x10^1 \frac{km}{hr}$$

$$C_y = A_y + B_y = 18 \frac{km}{hr} + -3.0 \frac{km}{hr} = 15 \frac{km}{hr}$$

Now we can get the magnitude and direction:

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(1.0x10^1 \frac{km}{hr})^2 + (15 \frac{km}{hr})^2} = \sqrt{1.0x10^2 \frac{km^2}{hr^2} + 230 \frac{km^2}{hr^2}} = \sqrt{330 \frac{km^2}{hr^2}} = 18 \frac{km}{hr}$$

Now for the angle:

$$\tan(\theta) = \frac{C_y}{C_x}$$

$$\tan(\theta) = \frac{15 \cancel{\frac{km}{hr}}}{1.0x10^1 \cancel{\frac{km}{hr}}}$$

$$\tan(\theta) = 1.5$$

$$\theta = \tan^{-1}(-10) = 56^{o}$$

Since the x-component is positive and the y-component is positive it is in quadrant I, so we do nothing to the angle. That's the actual velocity, but it is not the answer. The answer asks for displacement, which we can calculate, since we know the time. However, we do have to convert the time to hours, which I will assume you can do. It is 1.95 hr.

$$\Delta\mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 = \left(18 \frac{km}{hr}\right) \cdot (1.95\ hr) = 35\ km$$

3. The x-component is 180 m/s, and the y-component is -110 m/s. We need to know the initial x- and y-components.

$$\mathbf{v}_{0x} = \mathrm{v} \cdot \cos(\theta) = \left(213\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot \cos(32^{\mathrm{o}}) = 180\frac{\mathrm{m}}{\mathrm{s}}$$

$$\mathbf{v}_{0y} = \mathrm{v} \cdot \sin(\theta) = \left(213\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot \sin(32^{\mathrm{o}}) = 110\frac{\mathrm{m}}{\mathrm{s}}$$

The x-component never changes, and at the height from which it was released, the y-component is the negative of the initial y-component.

4. Both would have smaller magnitudes, because air resistance fights motion. You can also note that the reduction in the x-component will be larger.

5. The maximum height is 140 m. It takes 5.4 s to reach that heigh and 11 s to return to the height from which it was launched. The maximum height depends only on the y-component of the initial velocity.

$$\mathbf{v}_{0y} = \mathrm{v} \cdot \sin(\theta) = \left(54\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot \sin(77^{\mathrm{o}}) = 53\frac{\mathrm{m}}{\mathrm{s}}$$

We know **a** = -9.81 $\mathrm{m/s^2}$, and at the maximum height, the y-component of the velocity is zero. Thus:

$$\mathbf{v}_{\mathrm{f}}^{2} = \mathbf{v}_{0}^{2} + 2\mathbf{a} \cdot \Delta\mathbf{x}$$

$$0 = (53\frac{\mathrm{m}}{\mathrm{s}})^2 + 2(-9.81\frac{\mathrm{m}}{\mathrm{s}^2}) \cdot \Delta\mathbf{x}$$

$$\Delta\mathbf{x} = \frac{-(53\frac{\mathrm{m}}{\mathrm{s}})^2}{2 \cdot (-9.81\frac{\mathrm{m}}{\mathrm{s}^2})} = \frac{2{,}809\frac{\mathrm{m}^{\cancel{2}}}{\cancel{\mathrm{s}^2}}}{2 \cdot (9.81\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}^2}})} = 140\ \mathrm{m}$$

This still depends only on the y-component.

$$\mathbf{v}_{\mathrm{f}} = \mathbf{v}_0 + \mathbf{a} \cdot \mathrm{t}$$

$$0 = 53\frac{\mathrm{m}}{\mathrm{s}} + (-9.81\frac{\mathrm{m}}{\mathrm{s}^2}) \cdot \mathrm{t}$$

$$\mathrm{t} = \frac{53\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}}}}{9.81\frac{\cancel{\mathrm{m}}}{\mathrm{s}^{\cancel{2}}}} = 5.4\ \mathrm{s}$$

It takes twice that long (11 s) to get back to the height from which it was launched. Note that when multiplying by 2, the 2 is exact, but there are only two significant figures in 5.4, so the answer must be rounded to two significant figures.

6. The range is 7,400 m. The range is given by the range equation, since the final height is equal to the height from which it was fired.

$$\text{range} = \frac{\mathrm{v}^2 \cdot \sin(2\theta)}{\mathrm{g}} = \frac{(271\frac{\mathrm{m}}{\mathrm{s}})^2 \cdot \sin(2 \cdot 49^{\mathrm{o}})}{9.81\frac{\mathrm{m}}{\mathrm{s}^2}} = \frac{73{,}441\frac{\mathrm{m}^{\cancel{2}}}{\cancel{\mathrm{s}^2}} \cdot \sin(98^{\mathrm{o}})}{9.81\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}^2}}} = 7{,}400\ \mathrm{m}$$

7. The rifle should be aimed at an angle of 0.070°. The range equation applies, since the final height is equal to the height from which it was fired.

$$\text{range} = \frac{v^2 \cdot \sin(2\theta)}{g}$$

$$155\ \text{m} = \frac{(790\frac{\text{m}}{\text{s}})^2 \cdot \sin(2\theta)}{9.81\frac{\text{m}}{\text{s}^2}}$$

$$\sin(2\theta) = \frac{(155\ \cancel{\text{m}}) \cdot (9.81\frac{\cancel{\text{m}}}{\cancel{\text{s}^2}})}{(790\frac{\cancel{\text{m}}}{\cancel{\text{s}}})^2}$$

$$\sin(2\theta) = 0.002436\ldots$$

$$2\theta = \sin^{-2}(0.002436\ldots)$$

$$2\theta = 0.14^{\text{o}}$$

$$\theta = 0.070^{\text{o}}$$

8. It is 18 m above the height from which the ball was thrown. The question asks about the y-dimension, which probably means we should start with the x-dimension:

$$\mathbf{v}_{0x} = v \cdot \cos(\theta) = \left(37\frac{\text{m}}{\text{s}}\right) \cdot \cos(75^{\text{o}}) = 9.6\frac{\text{m}}{\text{s}}$$

Now we can use Equation (2.16) to determine the time:

$$\Delta\mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$5.2\ \text{m} = (9.6\frac{\text{m}}{\text{s}}) \cdot t + \frac{1}{2}(0) \cdot t^2$$

$$t = \frac{5.2\ \cancel{\text{m}}}{9.6\frac{\cancel{\text{m}}}{\text{s}}} = 0.54\ \text{s}$$

We can use that to determine its displacement in the y-dimension, but we first have to get the initial velocity in the y-dimension.

$$\mathbf{v}_{0y} = v \cdot \sin(\theta) = \left(37\frac{\text{m}}{\text{s}}\right) \cdot \sin(75^{\text{o}}) = 36\frac{\text{m}}{\text{s}}$$

We can now use Equation (2.16).

$$\Delta\mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 = \left(36\frac{\text{m}}{\cancel{\text{s}}}\right) \cdot (0.54\ \cancel{\text{s}}) + \frac{1}{2}\left(-9.81\frac{\text{m}}{\cancel{\text{s}^2}}\right)(0.54\ \cancel{\text{s}})^2 = 19\ \text{m} - 1.4\ \text{m} = 18\ \text{m}$$

9. It will drop 0.53 m. The question asks about the y-dimension, which probably means we should start with the x-dimension:

$$\mathbf{v}_{0x} = v \cdot \cos(\theta) = \left(750\frac{\text{m}}{\text{s}}\right) \cdot \cos(0^{\text{o}}) = 750\frac{\text{m}}{\text{s}}$$

Now we can use Equation (2.16) to determine the time:

$$\Delta\mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$250\ \mathrm{m} = (750\frac{\mathrm{m}}{\mathrm{s}}) \cdot \mathrm{t} + \frac{1}{2}(0) \cdot \mathrm{t}^2$$

$$\mathrm{t} = \frac{250\ \cancel{\mathrm{m}}}{750\frac{\cancel{\mathrm{m}}}{\mathrm{s}}} = 0.33\ \mathrm{s}$$

We can use that to determine its displacement in the y-dimension, but we first have to get the initial velocity in the y-dimension.

$$\mathbf{v}_{0y} = \mathrm{v} \cdot \sin(\theta) = \left(750\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot \sin(0^{\mathrm{o}}) = 0$$

We can now use Equation (2.16).

$$\Delta\mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 = \left(0\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot (0.33\ \mathrm{s}) + \frac{1}{2}\left(-9.81\frac{\mathrm{m}}{\cancel{\mathrm{s}^2}}\right)(0.33\ \cancel{\mathrm{s}})^2 = -0.53\ \mathrm{m}$$

10. It is 2.0x10[1] m to the target. The question asks about the x-dimension, so we should probably start with the y-dimension.

$$\mathbf{v}_{0y} = \mathrm{v} \cdot \sin(\theta) = \left(110\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot \sin(0^{\mathrm{o}}) = 0$$

Now we can use Equation (2.16) to determine the time, but we must convert the distance to meters to make it consistent with the units on acceleration:

$$\Delta\mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$0.170\ \mathrm{m} = (0\frac{\mathrm{m}}{\mathrm{s}}) \cdot \mathrm{t} + \frac{1}{2}(9.81\frac{\mathrm{m}}{\mathrm{s}^2}) \cdot \mathrm{t}^2$$

$$\mathrm{t} = \sqrt{\frac{2 \cdot 0.170\ \cancel{\mathrm{m}}}{9.81\frac{\cancel{\mathrm{m}}}{\mathrm{s}^2}}} = 0.186\ \mathrm{s}$$

We can now figure out how far it will travel in the x-dimension during that time. The x-component of the velocity is zero:

$$\mathbf{v}_{0x} = \mathrm{v} \cdot \cos(\theta) = \left(110\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot \cos(0^{\mathrm{o}}) = 110\frac{\mathrm{m}}{\mathrm{s}}$$

Now we can use Equation (2.16):

$$\Delta\mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 = \left(110\frac{\mathrm{m}}{\cancel{\mathrm{s}}}\right) \cdot (0.186\ \cancel{\mathrm{s}}) + \frac{1}{2}(0) \cdot (0.186\ \mathrm{s})^2 = 2.0 \times 10^1\ \mathrm{m}$$