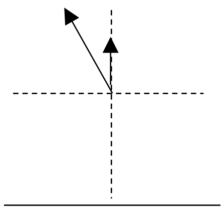
Solutions to the Extra Practice Problems for Chapter 4

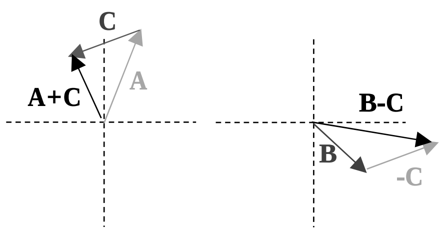
1. a. The length is not relevant, since there is nothing to compare it to. Thus, any length could represent 12 N. However, the angle is between 90° and 180°, so the vector is in quadrant II, but it is closer to the positive y-axis, since it is closer to 90° than 180°.

2.



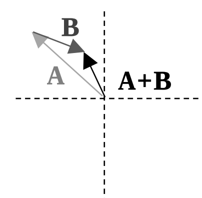
The length will be half of the 12-N vector, and the direction will be straight along the y-axis, because that represents 90°

3.



To add the vectors, we put the tail of the second one at the head of the first, and then we connect the tail of the first to the head of the second. To subtract, we take the negative of the second and then add it to the first.

4. a. The first vector (**A**) is midway between the positive y-axis (90°) and the negative x-axis (180°). The second vector (**B**) is half as long as the first, and it is between the negative y-axis (270°) and the positive x-axis (360°). However, it is closer to the positive x-axis Thus, the two vectors look something like the drawing on the right. When they add, they make a vector that is pointing up and to the left, but closer to the positive y-axis than the negative x-axis.



5. The x-component is -11.3 m/s and the y-component is -6.80 m/s. Equations (4.1) and (4.2) give us the components.

$$A_y = A \cdot \sin(\theta) = \left(13.2\ \frac{m}{s}\right) \cdot \sin(211^o) = -6.80\ \frac{m}{s}$$

$$A_x = A \cdot \cos(\theta) = \left(13.2\ \frac{m}{s}\right) \cdot \cos(211^o) = \ -11.3\ \frac{m}{s}$$

6. The displacement is 4.90 km at 302.2°. Remember, due east is along the positive x-axis, so the x-component is 2.61 km. Due south is along the negative y-axis, so the y-component is -4.15 km. We can get the magnitude from Equation (4.4)

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2.61\text{ km})^2 + (-4.15\text{ km})^2} = \sqrt{6.81\text{ km}^2 + 17.2\text{ km}^2} = \sqrt{24.0\text{ km}^2} = 4.90\text{ km}$$

Now we get the angle with Equation (4.3):

$$\tan(\theta) = \frac{A_y}{A_x}$$

$$\tan(\theta) = \frac{-4.15\ \cancel{\text{km}}}{2.61\ \cancel{\text{km}}}$$

$$\tan(\theta) = \ -1.5900\ ...$$

$$\theta = \ \tan^{-1}(-1.5900\ ...) = \ -57.8^o$$

Since the x-component is positive, that means it is right of the origin. Since the y-component is negative, it is below. To the right and below is the quadrant IV, so we must add exactly 360°:

$$\theta = -57.8^{o} + 360^{o} = 302.2^{o}$$

7. <u>The actual velocity is 17.5 km/hr at 129.0°</u>. The actual velocity (**C**) will be the sum of the ship's velocity (**A**) and the current's (**B**).

$$A_x = A \cdot \cos(\theta) = \left(21.1 \frac{km}{hr}\right) \cdot \cos(117^{o}) = -9.58 \frac{km}{hr}$$

$$A_y = A \cdot \sin(\theta) = \left(21.1 \frac{km}{hr}\right) \cdot \sin(117^{o}) = 18.8 \frac{km}{hr}$$

$$B_x = B \cdot \cos(\theta) = \left(5.4 \frac{km}{hr}\right) \cdot \cos(255^{o}) = -1.4 \frac{km}{hr}$$

$$B_y = B \cdot \sin(\theta) = \left(5.4 \frac{km}{hr}\right) \cdot \sin(255^{o}) = -5.2 \frac{km}{hr}$$

Now we can add the x-components and y-components. The results will be the x- and y-components of the vector sum:

$$C_x = A_x + B_x = -9.58 \frac{km}{hr} + -1.4 \frac{km}{hr} = -11.0 \frac{km}{hr}$$

$$C_y = A_y + B_y = 18.8 \frac{km}{hr} + -5.2 \frac{km}{hr} = 13.6 \frac{km}{hr}$$

Now we can get the magnitude and direction:

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{\left(-11.0 \frac{km}{hr}\right)^2 + \left(13.6 \frac{km}{hr}\right)^2} = \sqrt{121 \frac{km^2}{hr^2} + 185 \frac{km^2}{hr^2}} = \sqrt{306 \frac{km^2}{hr^2}} = 17.5 \frac{km}{hr}$$

Now for the angle:

$$\tan(\theta) = \frac{C_y}{C_x}$$

$$\tan(\theta) = \frac{13.6 \, \cancel{\frac{km}{hr}}}{-11.0 \, \cancel{\frac{km}{hr}}}$$

$$\tan(\theta) = -1.236 \ldots$$

$$\theta = \tan^{-1}(-1.236 \ldots) = -51.0^{o}$$

Since the x-component is negative and the y-component is positive, it is in quadrant II and we add exactly 180° to the angle

$$\theta = -51.0^{o} + 180^{o} = 129.0^{o}$$

8. The sum is <u>1.7 m at 45°</u>. First we break them down into their components:

$$A_x = A \cdot \cos(\theta) = (3.26 \text{ m}) \cdot \cos(241^{o}) = -1.58 \text{ m}$$

$$A_y = A \cdot \sin(\theta) = (3.26 \text{ m}) \cdot \sin(241^{o}) = -2.85 \text{ m}$$

$$B_x = B \cdot \cos(\theta) = (4.89 \text{ m}) \cdot \cos(55^{o}) = 2.8 \text{ m}$$

$$B_y = B \cdot \sin(\theta) = (4.89\text{ m}) \cdot \sin(55^o) = 4.0\text{ m}$$

Now we can add the x-components and y-components. The results will be the x- and y-components of the vector sum:

$$C_x = A_x + B_x = -1.58\text{ m} + 2.8\text{ m} = 1.2\text{ m}$$

$$C_y = A_y + B_y = -2.85\text{ m} + 4.0\text{ m} = 1.2\text{ m}$$

Now we can get the magnitude and direction:

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(1.2\text{ m})^2 + (1.2\text{ m})^2} = \sqrt{1.4\text{ m}^2 + 1.4\text{ m}^2} = \sqrt{2.8\text{ m}^2} = 1.7\text{ m}$$

Now for the angle:

$$\tan(\theta) = \frac{C_y}{C_x}$$

$$\tan(\theta) = \frac{1.2\text{ \sout{m}}}{1.2\text{ \sout{m}}}$$

$$\tan(\theta) = 1.0$$

$$\theta = \tan^{-1}(1.0) = 45^o$$

Since the x- and y-components are positive, the vector is in quadrant I, so that is the physics angle.

9. <u>The engines must give the airplane a velocity of 607 km/hr at 36.8°</u>. Calling the velocity given by the engines vector **A**, the velocity of the wind vector **B**, and the sum vector **C**, we need to find **A**.

$$C_x = C \cdot \cos(\theta) = \left(626\frac{\text{km}}{\text{hr}}\right) \cdot \cos(38.0^o) = 493\ \frac{\text{km}}{\text{hr}}$$

$$C_y = C \cdot \sin(\theta) = \left(626\frac{\text{km}}{\text{hr}}\right) \cdot \sin(38.0^o) = 385\ \frac{\text{km}}{\text{hr}}$$

When the x- and y-components of **A** and **B** are added, they must equal the numbers above, so let's get the components of **B**:

$$B_x = B \cdot \cos(\theta) = \left(22\frac{\text{km}}{\text{hr}}\right) \cdot \cos(75.0^o) = 5.7\ \frac{\text{km}}{\text{hr}}$$

$$B_y = B \cdot \sin(\theta) = \left(22\frac{\text{km}}{\text{hr}}\right) \cdot \sin(75.0^o) = 21\ \frac{\text{km}}{\text{hr}}$$

Now we can figure out the x- and y-components of **A**:

$$C_x = A_x + B_x$$

$$493\ \frac{\text{km}}{\text{hr}} = A_x + 5.7\frac{\text{km}}{\text{hr}}$$

$$A_x = 493\ \frac{\text{km}}{\text{hr}} - 5.7\frac{\text{km}}{\text{hr}} = 487\ \frac{\text{km}}{\text{hr}}$$

We can figure out the y-component that **A** must have to make this work:

$$C_y = A_y + B_y$$

$$385\ \frac{\text{km}}{\text{hr}} = A_y + 21\ \frac{\text{km}}{\text{hr}}$$

$$A_y = 385\ \frac{\text{km}}{\text{hr}} - 21\ \frac{\text{km}}{\text{hr}} = 364\ \frac{\text{km}}{\text{hr}}$$

We can now use those components to determine the magnitude and direction of the vector:

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(487\frac{\text{km}}{\text{hr}}\right)^2 + \left(364\frac{\text{km}}{\text{hr}}\right)^2} = \sqrt{237{,}000\frac{\text{km}^2}{\text{hr}^2} + 132{,}000\frac{\text{km}^2}{\text{hr}^2}} = \sqrt{369{,}000\frac{\text{km}^2}{\text{hr}^2}} = 607\frac{\text{km}}{\text{hr}}$$

Now for the angle:

$$\tan(\theta) = \frac{A_y}{A_x}$$

$$\tan(\theta) = \frac{364\ \cancel{\frac{\text{km}}{\text{hr}}}}{487\ \cancel{\frac{\text{km}}{\text{hr}}}}$$

$$\tan(\theta) = 0.7474\ldots$$

$$\theta = \tan^{-1}(0.7474\ldots) = 36.8^{\text{o}}$$

This is the physics angle, since both the x- and y-components are positive. That means we are in quadrant I and don't need to do anything to the angle the calculator gives us.

10. The current's velocity is <u>4.91 km/hr at 260º</u>. Calling the velocity given by the engines vector **A**, the velocity of the current vector **B**, and the sum vector **C**, we need to find **B**.

$$C_x = C \cdot \cos(\theta) = \left(11.8\frac{\text{km}}{\text{hr}}\right) \cdot \cos(165^{\text{o}}) = -11.4\frac{\text{km}}{\text{hr}}$$

$$C_y = C \cdot \sin(\theta) = \left(11.8\ \frac{\text{km}}{\text{hr}}\right) \cdot \sin(165^{\text{o}}) = 3.05\ \frac{\text{km}}{\text{hr}}$$

When the x- and y-components of **A** and **B** are added, they must equal the numbers above, so let's get the components of **A**:

$$A_x = A \cdot \cos(\theta) = \left(13.1\frac{\text{km}}{\text{hr}}\right) \cdot \cos(143^{\text{o}}) = -10.5\ \frac{\text{km}}{\text{hr}}$$

$$A_y = A \cdot \sin(\theta) = \left(13.1\frac{\text{km}}{\text{hr}}\right) \cdot \sin(143^{\text{o}}) = 7.88\ \frac{\text{km}}{\text{hr}}$$

Now we can figure out the x- and y-components of **B**:

$$C_x = A_x + B_x$$

$$-11.4\ \frac{\text{km}}{\text{hr}} = -10.5\frac{\text{km}}{\text{hr}} + B_x$$

$$B_x = -11.4\frac{\text{km}}{\text{hr}} + 10.5\frac{\text{km}}{\text{hr}} = -0.9\ \frac{\text{km}}{\text{hr}}$$

We can figure out the y-component that **B** must have to make this work:

$$C_y = A_y + B_y$$

$$3.05\ \frac{km}{hr} = 7.88\ \frac{km}{hr} + B_y$$

$$B_y = 3.05\ \frac{km}{hr} - 7.88\ \frac{km}{hr} = -4.83\ \frac{km}{hr}$$

We can now use those components to determine the magnitude and direction of the vector:

$$B = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(-0.9\frac{km}{hr}\right)^2 + \left(-4.83\frac{km}{hr}\right)^2} = \sqrt{0.8\frac{km^2}{hr^2} + 23.3\frac{km^2}{hr^2}} = \sqrt{24.1\ \frac{km^2}{hr^2}} = 4.91\frac{km}{hr}$$

Since 2 has only one significant figure, its square has only one. That makes its only significant figure in the ones place, so the sum must have its last significant figure in the ones place. That gives only one significant figure for the square root. Now for the angle:

$$\tan(\theta) = \frac{A_y}{A_x}$$

$$\tan(\theta) = \frac{-4.83\ \frac{\cancel{km}}{\cancel{hr}}}{-0.9\ \frac{\cancel{km}}{\cancel{hr}}}$$

$$\tan(\theta) = 5.3666\ldots$$

$$\theta = \tan^{-1}(5.3666\ldots) = 80^o$$

The x-component is negative, and the y-component is negative, so this is quadrant III, where we add exactly 180°.